

# NEUTRINOS AND BIG BANG NUCLEOSYNTHESIS

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The role of neutrinos in big bang nucleosynthesis is discussed. The bounds on the number of neutrino families, neutrino degeneracy, parameters of neutrino oscillations are presented. A model of chemically inhomogeneous, while energetically smooth, universe created by inhomogeneous cosmological neutrino asymmetry is described. Nucleosynthesis limits on production of right-handed neutrinos are considered.

## 1 Introduction

Big Bang Nucleosynthesis (BBN) is one of the strongest evidences in favor of the Standard Hot Cosmological Model. The theory predicts abundances of light elements  $^2\text{H}$ ,  $^3\text{He}$ ,  $^4\text{He}$ , and  $^7\text{Li}$  which span 9 orders of magnitude in a good agreement with observations. Theoretical calculations are well defined and with the existing uncertainties in the cross-sections of the relevant nuclear reactions the accuracy of calculations is about 10% or, depending upon the element, is even better, especially for  $^4\text{He}$ . However, comparison of theoretical results with observations is not straightforward because the data are subject to poorly known evolutionary effects and systematic errors. Still, even with these uncertainties, BBN permits to eliminate many modifications of the standard model and to derive strong restrictions on properties of elementary particles, in particular, on neutrino.

In what follows I will briefly discuss physics of BBN and essential parameters that determine production of light elements (sec. 2). After that the role of neutrinos in BBN is described. The bound on the number of neutrino species is presented in section 3. In section 4 neutrino degeneracy and its impact on BBN are discussed. Neutrino oscillations and their possible influence on BBN are considered in sec. 5. Inhomogeneous BBN related to possible spatial variations of lepton asymmetry and producing large fluctuations of primordial abundances in different regions of the universe is discussed in sec. 6. In sec. 7 limits on some other neutrino properties (mass, magnetic moments, right-handed currents, etc) are presented.

The frameworks of the talk do not permit to give the complete list of references to the discussed problems, so I indicated only original and most recent papers. A history of the problems and much more detailed list of references can be found in the review <sup>1</sup>.

## 2 Physics of BBN

Building blocks for production of light nuclei were prepared in the universe when the temperature was about 1 MeV. The reactions

$$n + e^+ \leftrightarrow p + \bar{\nu}_e,$$

$$n + \nu_e \leftrightarrow p + e^- \quad (1)$$

maintained thermally equilibrium ratio of neutrons to protons

$$n/p = \exp(-\Delta m/T - \xi_e) \quad (2)$$

till the temperature  $T$  dropped down below  $T_f = 0.6 - 0.7$  MeV. Here  $\Delta m = 1.3$  MeV is the neutron-proton mass difference and  $\xi_e = \mu_e/T$  is the dimensionless chemical potential of electronic neutrinos. At temperature below  $T_f$  the reactions (1) became practically frozen, their rate  $\Gamma \sim G_F^2 T^5$  became much smaller than the universe expansion rate,  $H = \dot{a}/a \sim T^2$ , where  $G_F$  is the Fermi coupling constant,  $H$  is the Hubble parameter and  $a(t)$  is the cosmological scale factor. Correspondingly the ratio  $n/p$  would remain constant if neutrons were stable. Due to neutron decay with the life-time  $\tau_n = 888 \pm 2$  sec the  $n/p$ -ratio drops as  $\exp(-t/\tau_n)$ .

Formation of light elements through the chain of reactions  $p + n \rightarrow {}^2H + \gamma$ ,  $p + {}^2H \rightarrow {}^3He$ ,  $n + {}^2H \rightarrow {}^3H$ ,  ${}^3He + n \rightarrow {}^4He$ , etc started at  $T = T_{NS} = 60 - 70$  keV. The concrete value of  $T_{NS}$  depends upon baryon-to-photon ratio  $\eta = n_B/n_\gamma \approx 5 \cdot 10^{-10}$  (found from BBN itself and now from the angular spectrum of CMBR as well) and the nuclear binding energies. The temperature  $T_{NS}$  is so much smaller than the nuclear binding energy because of very large number of cosmic photons per one baryon. At  $T = T_{NS}$  all neutrons were quickly captured and no free neutrons remained in the plasma. Since  ${}^4He$  has the largest binding energy practically all the neutrons, that survived to the moment when  $T = T_{NS}$ , ended their lives in  ${}^4He$  nuclei.

The universe cooling rate can be obtained if one compares the cosmological energy density expressed in terms of the Hubble parameter and the energy density of thermally equilibrium plasma:

$$\rho_{tot} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{\pi^2}{30} g_* T^4 \quad (3)$$

where  $g_*$  counts the number of relativistic species in the primeval plasma; it includes 2 from photons,  $4 \cdot (7/8)$  from  $e^\pm$ -pairs, and  $2 \cdot 3 \cdot (7/8)$  from three light neutrino families:

$$g_* = 10.75 + \frac{7}{4} (N_\nu - 3), \quad (4)$$

and the last term describes contribution of some non-standard energy. The latter could be additional neutrino species, possibly sterile, non-coupled to  $W$  and  $Z$  bosons, or new abundant massive particles, or some unknown form of dark energy (e.g. vacuum energy, or in other words, cosmological constant). In the last two cases equation of state of these unknown forms of matter would be different from relativistic equation of state  $p = \rho/3$  valid for light neutrinos and thus the parameter  $N_\nu$  could be a function of time. Moreover, the effect of this parameter on production of different elements may differ from the effect induced by additional neutrinos.

As follows from eq. (3) the universe cooling rate is determined by the expression:

$$\left( \frac{t}{\text{sec}} \right) \left( \frac{T}{\text{MeV}} \right)^2 = 0.74 \left( \frac{10.75}{g_*} \right)^2. \quad (5)$$

Since the cooling rate depends upon the effective number of particle species, BBN is sensitive to any form of energy present in the cosmic plasma in the range of temperatures from a few MeV down to 60 keV.

As one can see from the discussion above primordial abundances of light elements are the functions of:

1. weak interaction rate, which determines the moment when the reactions of neutron-proton transformation freezes; this rates is determined by the neutron life-time;
2. cosmological energy density parametrized as  $(N_\nu - 3)$ ;
3. number density of baryons,  $\eta_{10} = 10^{10} n_B / n_\gamma$ ;
4. neutrino degeneracy, given by dimensionless chemical potentials,  $\xi_a = \mu_a / T$ , where  $a = e, \mu, \tau$ ;  $\xi_a$  remains constant during adiabatic expansion if corresponding leptonic charge is conserved.

### 3 Number of neutrino families

As we have already mentioned in the previous section, an addition of an extra neutrino family would change the freezing temperature of  $n/p$ -transformation (1). Comparing reaction rate and the rate of cosmological expansion one can find  $T_f \sim g_*^{-1/6}$ . So with rising number of species the number density of neutrons and correspondingly the mass fraction of produced  ${}^4\text{He}$  also rises. Another effect leading in the same direction is that the time when the nucleosynthesis temperature,  $T_{NS}$  is reached also depends upon  $g_*$ ; according to eq. (5)  $t_{NS} \sim g_*^{-1/2}$ , so with larger  $g_*$  less neutrons would decay before the onset of nucleosynthesis and more  ${}^4\text{He}$  would be produced. Both these effects give rise to the increase of mass fraction of primordial helium-4 by approximately 5% if one extra neutrino family is added.

The dependence of helium production on the number of neutrino families was first mentioned in ref. <sup>2</sup> and a little later in ref. <sup>3</sup>. A detailed investigation was done in the papers <sup>4,5</sup>.

Comparing observational data with the theory one can deduce an upper limit on the number of additional neutrino families,  $\Delta N_\nu = N_\nu - 3$ . According to the numerous literature on the subject, this limit oscillates between 2 and 0.1. The recent analysis of ref. <sup>6</sup> gives  $N_\nu = 1.8 - 3.9$ .

It is usually assumed that neutrinos have the equilibrium spectrum:

$$f_\nu = [\exp(E/T - \xi) + 1]^{-1} \quad (6)$$

and the corresponding energy density of neutrinos plus antineutrinos is

$$\begin{aligned} \rho_\nu + \rho_{\bar{\nu}} &= \frac{1}{2\pi^2} \int_0^\infty dp p^3 \left[ \frac{1}{e^{E/T-\xi} + 1} + \frac{1}{e^{E/T+\xi} + 1} \right] \\ &= \frac{7}{8} \frac{\pi^2 T^4}{15} \left[ 1 + \frac{30}{7} \left( \frac{\xi}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi}{\pi} \right)^4 \right] \end{aligned} \quad (7)$$

The limit on the number of neutrino families presented above is obtained under assumption that neutrinos are not degenerate, i.e.  $\xi = 0$ . This limit is also valid for non-equilibrium  $\nu_\mu$  and  $\nu_\tau$  with the same energy density as the equilibrium ones. This is not so for electronic neutrinos because the latter effect the  $n/p$ -ratio not only by their energy density but also more directly by their spectrum, since they took part in the reactions (1). Surprisingly the spectrum of all neutrinos noticeably deviates from the equilibrium one, though they may be exactly massless. This deviation is induced by different temperatures of electrons and neutrinos due to  $e^+e^-$ -annihilation after neutrino decoupling<sup>7,8</sup>. According to analytical estimate of ref.<sup>7</sup> the spectral distortion has the form:

$$\frac{\delta f_{\nu_e}}{f_{\nu_e}} \approx 3 \cdot 10^{-4} \frac{E}{T} \left( \frac{11E}{4T} - 3 \right) \quad (8)$$

where  $\delta f = f - f^{(eq)}$ . The distortion of the spectra of  $\nu_\mu$  and  $\nu_\tau$  is approximately twice weaker. The shift of helium-4 mass fraction due to this neutrino heating by the residual  $e^+e^-$ -annihilation is only a few  $\times 10^{-4}$ , though the total neutrino energy density becomes larger than the standard one by about 3%<sup>9</sup>. There is another effect of the similar magnitude and sign<sup>10</sup>, namely finite-temperature electromagnetic corrections to the energy density of  $\gamma e^+e^-$ -plasma. It adds 0.01 effective number of extra neutrino species.

#### 4 Lepton Asymmetry

If the number density of particles is different from the number density of antiparticles, i.e. charge asymmetry is non-vanishing, it is described by a non-zero chemical potential,  $\xi$  (6). Of course this description is valid only in the case of kinetic equilibrium when the distribution in energy has the canonical form dictated by kinetic equation with strong elastic scattering term. Fast annihilation processes imply also opposite values of chemical potentials for particles and antiparticles:  $\bar{\xi} = -\xi$ .

According to eq. (6) energy density of degenerate neutrinos in thermal equilibrium is larger than the energy density of non-degenerate ones. The role of degenerate  $\nu_\mu$  and  $\nu_\tau$  in BBN is simply to increase the total energy density which corresponds to

$$\Delta N_\nu(\xi) = \frac{15}{7} \sum_{a=\mu,\tau} \left[ \left( \frac{\xi_a}{\pi} \right)^4 + 2 \left( \frac{\xi_a}{\pi} \right)^2 \right] \quad (9)$$

The bound on  $\Delta N_\nu$  presented above can be translated to the bound on the magnitude of the chemical potentials. In particular, if  $\Delta N_\nu < 1$  then  $|\xi_{\mu,\tau}| < 1.5$ .

The bound on the value of  $|\xi_e|$  is much stronger because degeneracy of electronic neutrinos exponentially shifts  $n/p$ -ratio, see eq. (2). The BBN bounds on chemical potentials would be somewhat weaker if combined variation of all chemical potentials is allowed. In this case a large value of  $|\xi_{\mu,\tau}|$  may be compensated by a relatively small and positive  $\xi_e$ <sup>11</sup>. Recent analysis of the work<sup>12</sup> based on additional information extracted from the measurements of angular fluctuations of CMBR permits to obtain the limits:

$$-0.01 < \xi_{\nu_e} < 0.2, \quad |\xi_{\nu_\mu, \nu_\tau}| < 2.6 \quad (10)$$

under assumptions that the primordial fraction of deuterium is  $D/H = (3.0 \pm 0.4) \cdot 10^{-5}$ . More details and references can be found in the review <sup>1</sup>.

These results can be further strengthened if there are neutrino oscillations that mix  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ . In this case muonic or tauonic asymmetries would be transformed through oscillations into electronic asymmetry. This problem was analyzed recently in ref. <sup>13</sup> where it was shown that for large mixing angle solution for the solar neutrino anomaly and for  $(\nu_\mu - \nu_\tau)$ -mixing deduced from the atmospheric neutrino anomaly the bounds on chemical potentials of all flavors are approximately

$$|\xi_a| < 0.1 \quad (11)$$

## 5 Neutrino Oscillations and BBN

Effects of neutrino oscillations on BBN are very much different if only active neutrinos are mixed or mixing is allowed between active and hypothetical sterile neutrinos. In the first case oscillations do not create any deviation from the standard BBN results if neutrinos are in thermal equilibrium with vanishing chemical potentials. Indeed, in this case oscillations would not lead to any modification of the standard distribution functions of neutrinos and abundances of light elements would remain the same as they were without oscillations. A noticeable effect would arise if neutrinos are strongly degenerate and different lepton asymmetries would be redistributed by oscillations as discussed in the previous section.

Physics is much more interesting if there are oscillations between active and sterile neutrinos. One evident effect is that oscillations would create additional neutrino species leading to  $N_\nu > 3$ . Second, oscillations may distort spectrum of electronic neutrinos. The sign of the effect may be both positive or negative depending upon the form of spectral distortion. Third, in the case of MSW-resonance oscillations between  $\nu_e$  and  $\nu_s$  could be more efficient than oscillation between antineutrinos or vice versa. This would give rise to generation of lepton asymmetry in the sector of active neutrinos and, in particular, of electronic charge asymmetry which would have a strong impact on BBN.

Excitation of additional, sterile, degrees of freedom at BBN by neutrino oscillations was considered in many papers starting from 1990 (a large list of references can be found in <sup>1</sup>). A recent bound on the mixing parameters between  $\nu_s$  and  $\nu_e$  and  $\nu_s$  and  $\nu_\mu$  or  $\nu_\tau$  respectively reads <sup>14</sup>:

$$(\delta m_{\nu_e \nu_s}^2 / \text{eV}^2) \sin^4 2\theta_{vac}^{\nu_e \nu_s} = 3.16 \cdot 10^{-5} (g_*(T_{prod}^{\nu_s}) / 10.75)^3 (\Delta N_\nu)^2 \quad (12)$$

$$(\delta m_{\nu_\mu \nu_s}^2 / \text{eV}^2) \sin^4 2\theta_{vac}^{\nu_\mu \nu_s} = 1.74 \cdot 10^{-5} (g_*(T_{prod}^{\nu_s}) / 10.75)^3 (\Delta N_\nu)^2 \quad (13)$$

where the number of relativistic degrees of freedom  $g_*$  is taken at the temperature  $T_{prod}^{\nu_s}$  at which sterile neutrinos are effectively produced:

$$T_{prod}^{\nu_s} = (12, 15) (3/y)^{1/3} (\delta m^2 / \text{eV}^2)^{1/6} \text{ MeV} \quad (14)$$

These bounds are valid only if  $\Delta N_\nu < 1$  and spectral distortion of  $\nu_e$  is neglected.

The impact on BBN of the distortion of the spectrum of  $\nu_e$  by oscillations was discussed e.g. in ref. <sup>15</sup>. According to this work an analytical fit to the bound on

the oscillation parameters that follows from the consideration of primordial  ${}^4\text{He}$  can be written as

$$\delta m^2 (\sin^2 2\theta)^4 \leq 1.5 \cdot 10^{-9} \text{ eV}^2, \text{ for } \delta m^2 < 10^{-7} \text{ eV}^2. \quad (15)$$

The situation is much more complicated for a larger mass difference. In particular, for the mass difference  $\delta m^2 \sim (1 - 100) \text{ eV}^2$  and a small vacuum mixing angle,  $\sin^2 2\theta < 10^{-3}$ , the resonance amplification of lepton asymmetry in the sector of active neutrinos can take place<sup>16</sup>. The effect was discussed in detail in the subsequent literature and now seems to be confirmed both numerically and analytically. The impact of this phenomenon on BBN could be quite strong but no simple analytical results have been presented. An example of calculation of the effective neutrino number  $\Delta N_\nu$  induced by the generation of asymmetry in  $\nu_e$  sector can be found in ref.<sup>17</sup>.

## 6 Spatial Variation of Primordial Abundances

An interesting phenomenon would arise if lepton asymmetries were inhomogeneous and large during BBN. In this case a large spatial variation of primordial abundances of light elements should take place. If the scale of variation is larger than the mixing scale (galactic), then the observed abundances of light elements would be different in different regions of the universe. A mechanism of generation of large and inhomogeneous lepton asymmetry together with a small and possibly homogeneous baryon asymmetry was considered long ago in ref.<sup>18</sup> in the frameworks of Affleck and Dine lepto/baryogenesis scenario. Another model of creation of large and inhomogeneous lepton asymmetry by resonance neutrino oscillations (similar to the discussed in the previous section) was suggested recently in the paper<sup>19</sup>.

To avoid large density perturbations generated by varying chemical potentials it was assumed<sup>20</sup> that there exists symmetry with respect to permutation of electron, muon and tauon asymmetries. In the simplest version of such model electron asymmetry is small over 2/3 of the sky but muon or tauon asymmetries are large (of order unity). Abundances of light elements are normal there, i.e. the mass fraction of primordial  ${}^4\text{He}$  is  $Y_p \approx 0.25$  and the deuterium-hydrogen ratio is  $D/H = 3 \cdot 10^{-5}$ . Over 1/6 of the sky where electron asymmetry is large and negative and other asymmetries are small the primordial abundances are high,  $Y_p \approx 0.5$  and  $D/H = 10 \cdot 10^{-5}$ , and over other 1/6 of the sky where electron asymmetry is large and positive the abundances are low,  $Y_p \approx 0.12$  and  $D/H = 1.5 \cdot 10^{-5}$ . In more complicated versions of the model the probability distribution of the regions with normal, high, and low primordial abundances could be different.

The characteristic scale of variation is not predicted by the model but from the upper limit on angular variation of the temperature of CMBR one may conclude that the scale should be larger than a few hundreds Mpc<sup>20</sup>. Still some features in the angular spectrum of CMBR may be observable, in particular, diffusion damping slope at high  $l$  could be different in different directions in the sky.

The model discussed above presents an example of cosmology with broken Copernicus Principle: the universe is energetically smooth but strongly chemically inhomogeneous.

## 7 Non-standard Neutrino Properties

If neutrinos are massive (with Dirac mass) or possess right-current interactions then in addition to the usual left-handed species right-handed neutrino states might be present at BBN. Kinematical excitation of right-handed neutrino states by their Dirac mass was considered in many papers (see the review <sup>1</sup>). The latest and the most accurate work <sup>21</sup> presents the limits:

$$\begin{aligned} m_{\nu_\mu} &\leq \begin{cases} 130 \text{ keV}, & T_{QCD} = 100 \text{ MeV} \\ 120 \text{ keV}, & T_{QCD} = 200 \text{ MeV} \end{cases} \\ m_{\nu_\tau} &\leq \begin{cases} 150 \text{ keV}, & T_{QCD} = 100 \text{ MeV} \\ 140 \text{ keV}, & T_{QCD} = 200 \text{ MeV}. \end{cases} \end{aligned} \quad (16)$$

if  $\Delta N_\nu < 0.3$ , while for  $\Delta N_\nu < 1.0$ , they are

$$\begin{aligned} m_{\nu_\mu} &\leq \begin{cases} 310 \text{ keV}, & T_{QCD} = 100 \text{ MeV} \\ 290 \text{ keV}, & T_{QCD} = 200 \text{ MeV} \end{cases} \\ m_{\nu_\tau} &\leq \begin{cases} 370 \text{ keV}, & T_{QCD} = 100 \text{ MeV} \\ 340 \text{ keV}, & T_{QCD} = 200 \text{ MeV}. \end{cases} \end{aligned} \quad (17)$$

These limits are much stronger than laboratory limits for  $\nu_\tau$  mass and comparable to the limit on  $\nu_\mu$  mass. They are applicable if the neutrino life-time is longer than the characteristic time of nucleosynthesis (of course, because of Gerstein-Zeldovich limit <sup>22</sup> the life-time should be shorter than the universe age). On the other hand, an interpretation of neutrino anomalies in terms of neutrino oscillations demands very small mass difference and together with the laboratory bound,  $m_{\nu_e} < 3 \text{ eV}$  (see ref. <sup>23</sup>), they lead to the masses of  $\nu_\mu$  and  $\nu_\tau$  much below the bounds (16,17).

If neutrinos, in addition to left-handed currents, are also coupled to right-handed ones, then the right-handed degrees of freedom could be present at BBN even if neutrinos are strictly massless. To avoid too many right-handed neutrinos the interactions with right-handed currents should be sufficiently weak and decouple at high temperatures such that the entropy dilution due to massive particle annihilation would diminish the number density of right-handed neutrinos down to a safe for BBN value. In other words, the mass of right-handed intermediate bosons should be sufficiently high. There is some disagreement in the literature about the exact value of the lower bound on their mass, but roughly speaking it should be larger than a few TeV (see review <sup>1</sup>).

Another source of production of right-handed neutrinos could be their magnetic moment. There are two possible types of processes in the early universe where neutrino spin flip due to magnetic moment might take place: first, direct particle reactions, either quasi-elastic scattering,  $e^\pm + \nu_L \rightarrow e^\pm + \nu_R$ , and annihilation,  $e^- + e^+ \rightarrow \nu_{L,R} + \bar{\nu}_{L,R}$ , or the plasmon decay,  $\gamma_{pl} \rightarrow \bar{\nu}_{L,R} + \nu_{R,L}$ . The second process is the classical spin rotation of neutrinos in large scale primordial magnetic fields that might have existed in the early universe. The former mechanism was first considered in ref. <sup>24</sup>, while the second one in refs. <sup>25,26</sup>.

Assuming that the extra number of neutrino species allowed by BBN is smaller than 1, the authors of the most recent paper <sup>27</sup> found the limit:

$$\mu_\nu < 2.9 \cdot 10^{-10} \mu_B \quad (18)$$

for the case of reaction produced neutrinos.

Potentially stronger limit can be obtained from consideration of neutrino spin-flip in cosmological magnetic fields at nucleosynthesis epoch. Unfortunately the result depends upon poorly known field strength in the early universe. With a reasonable assumption about the latter the upper bound on magnetic moment of neutrinos could be much stronger than (18), see discussion in the review <sup>1</sup>.

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